Using Rijndael-256 in AES Modes

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AES --- 25 years of competing designs and cryptanalysis
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Thanks to Kazuhiko Minematsu for many discussions. Part of the content was presented at DIAC 2013

NIST SP800-38 Series

- NIST AES standardization process (1997-2001)
- NIST modes development (2000-ongoing)
  - Standard modes for AES and TDES
  - SP 800-38A, B, C, D, E, F, G

NIST SP800-38 Series

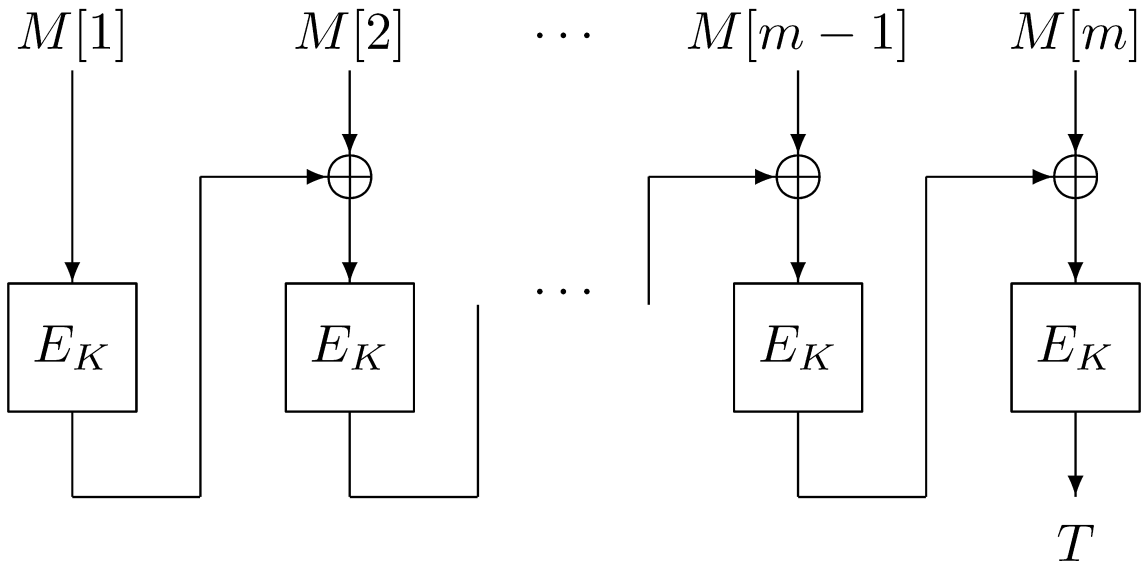
- SP 800-38A, Dec 2001, ECB, CBC, CFB, OFB, CTR
  - Oct 2010, Ciphertext stealing for CBC
- SP 800-38B, May 2005, CMAC
- SP 800-38C, May 2004, CCM
- SP 800-38D, Nov 2007, GCM and GMAC
- SP 800-38E, Jan 2010, XTS-AES
- SP 800-38F, Dec 2012, AES Key Wrap (KW) and AES KW with Padding (KWP)
- SP 800-38G, Mar 2016, FF1 and FF3, for format-preserving encryption

Outline

- AES-CMAC and Rijndael-256-CMAC
- AES-GCM and Rijndael-256-GCM

Development of CMAC

- CBC-MAC: a message authenticate code based on a block cipher



Development of CMAC

- Issues of the basic CBC-MAC and its variants (-2000)
  - does not cover variable-length input
    - Length-extension attack
  - does not cover arbitrarily input length
  - birthday bound security
  - key length
  - number of block cipher calls
  - …

Development of CMAC

- Development of AES (1997-2001) and NIST modes development (2000-ongoing) triggered a few proposals
- 2000-
  - XCBC [BR00]
  - RMAC [JJV02]
  - TMAC [KI03]
  - OMAC [IK03]
  - …

[BR00] Black and Rogaway. CBC MACs for arbitrary-length messages: the three-key construction. CRYPTO 2000.

[JJV02] Jaulmes, Joux, and Valette. On the security of randomized CBC-MAC beyond the birthday paradox limit: A new construction. FSE 2002.

[KI03] Kurosawa and Iwata. TMAC: Two-key CBC MAC. CT-RSA 2003.

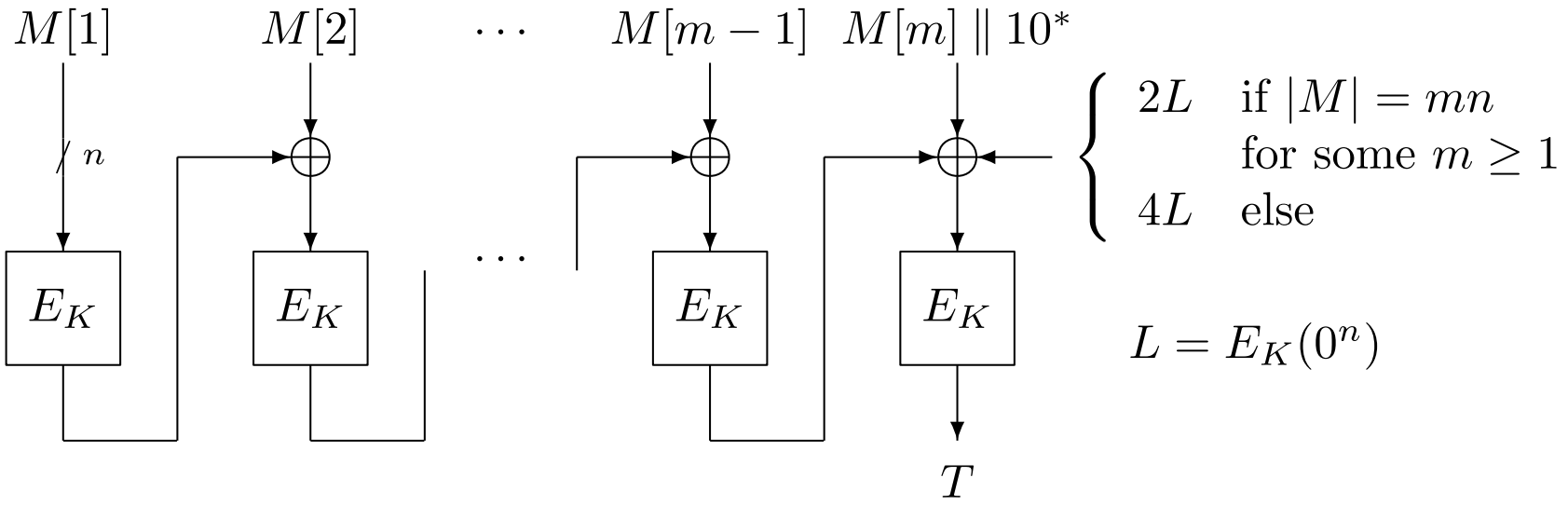
[IK03] Iwata and Kurosawa. OMAC: One-key CBC MAC. FSE 2003.

Development of CMAC

- NIST modes development (2000-ongoing)
- 6 MAC proposals (-2002)
  - OMAC, PMAC, RMAC, TMAC, XCBC, XECB
- NIST eventually selected OMAC in 2005 in NIST SP 800-38B, and named it CMAC for Cipher-based MAC
  - “The core of the CMAC algorithm is a variation of CBC-MAC that Black and Rogaway proposed and analyzed under the name XCBC in Ref. [2] …
    Iwata and Kurosawa proposed an improvement of XCBC and named the resulting algorithm One-Key CBC-MAC (OMAC) in Ref. [6] …
    The OMAC1 variation efficiently reduces the key size of XCBC. CMAC is equivalent to OMAC1.” [NIST SP 800-38B]

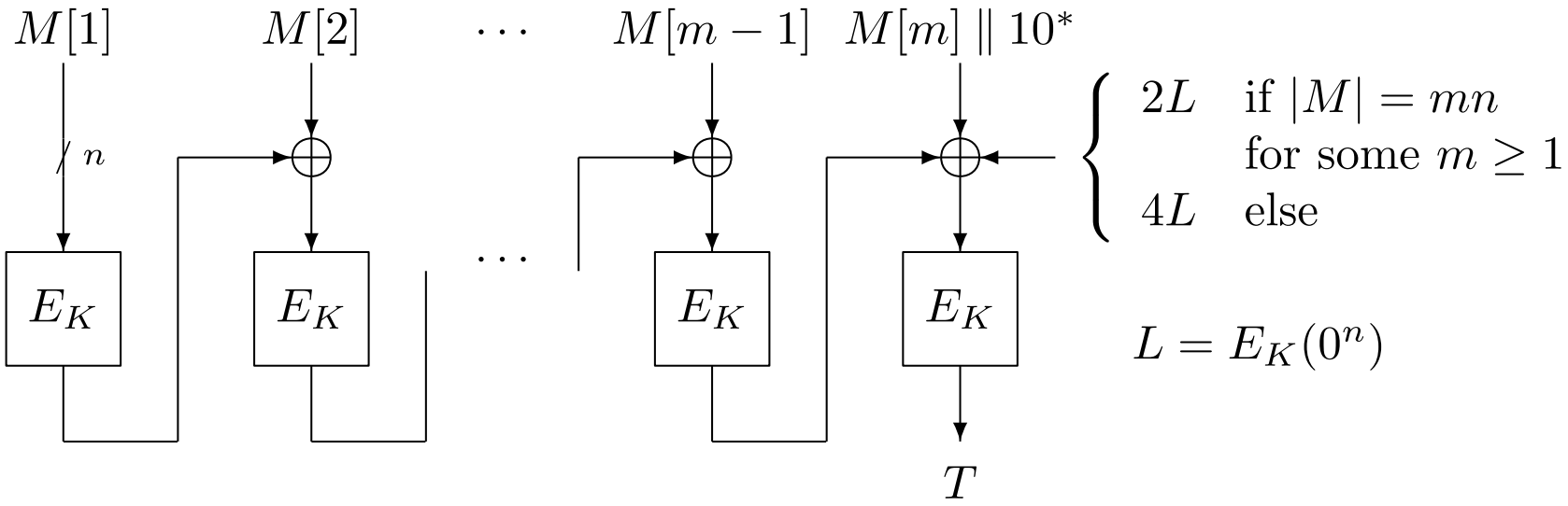
OMAC [IK03]

- Key $= K, k$ bits
- $1 + \lceil |M|/n \rceil$ block cipher calls



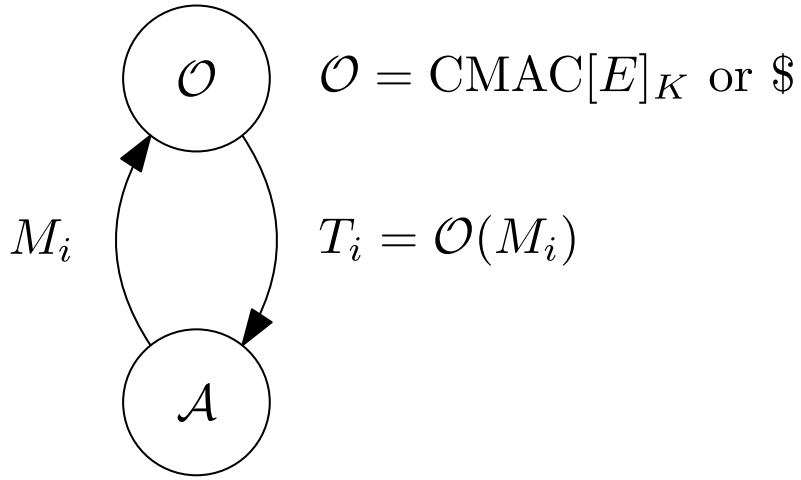
Proving the Security of CMAC

- Goal: CMAC ≈ random function
- $M[1] = 0^n$ implies L appears in the MAC computation
  - has to deal with the dependency



Proving the Security of CMAC

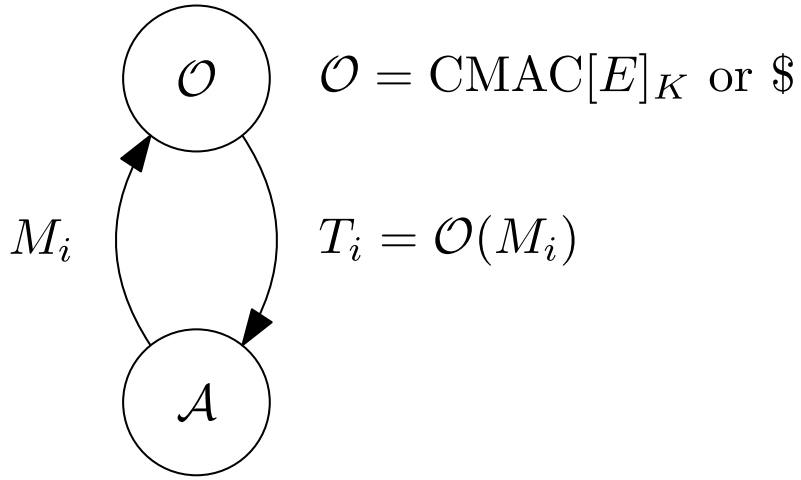
- Goal: CMAC ≈ random function
- Approach: $\mathrm{CMAC}[E]_K \approx \mathrm{CMAC}[P] \approx$ random function \$



Proving the Security of CMAC

- q: number of queries, σ: total number of blocks in all the queries, ℓ: the max block-length among the queries
  - $M_1 = M_1[1] \,||\, M_1[2] \,||\, M_1[3]$
  - $M_2 = M_2[1] \,||\, M_2[2]$
  - $M_3 = M_3[1] \,||\, M_3[2] \,||\, M_3[3] \,||\, M_3[4]$

    $\Rightarrow \quad q = 3, \sigma = 9, \ell = 4$
- Original security bound [IK03]: $O(\ell^2 q^2 \,/\, 2^n)$



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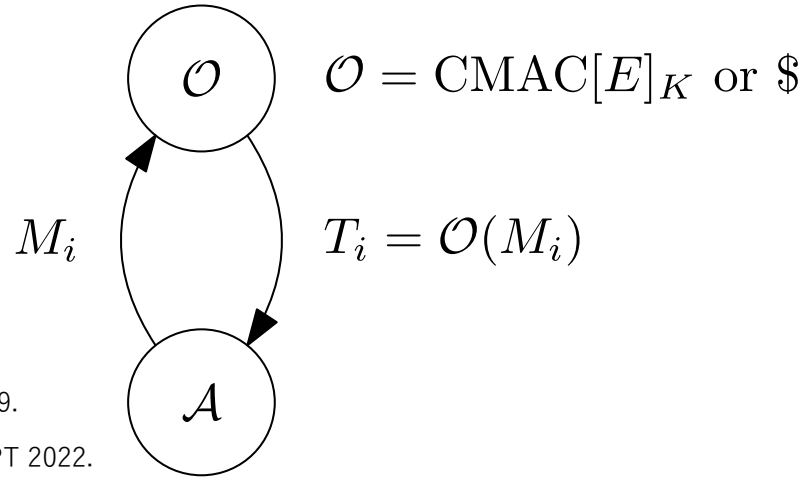
- Original security bound [IK03]: $O(\ell^2 q^2 \,/\, 2^n)$
- Bound in [IK04]: $O(\sigma^2 \,/\, 2^n)$
- Bound in [Nan09]: $O(q\sigma \,/\, 2^n)$ if $\ell < 2^{n/3}$
- Bound in [CJN22]: $O(q^2 / 2^n \;+\; q\ell^2 \,/\, 2^n)$



[Nan09] Nandi. Improved security analysis for OMAC as a pseudorandom function. J. Mathematical Cryptol., 2009.

[CJN22] Chattopadhyay, Jha, and Nandi. Towards Tight Security Bounds for OMAC, XCBC and TMAC. ASIACRYPT 2022.

Proving the Security of CMAC

- Bound in [IK04]: $O(\sigma^2 / 2^n)$

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathrm{CMAC}[E]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(\mathcal{A}') + \frac{4\sigma^2}{2^n}$$

- If E = AES

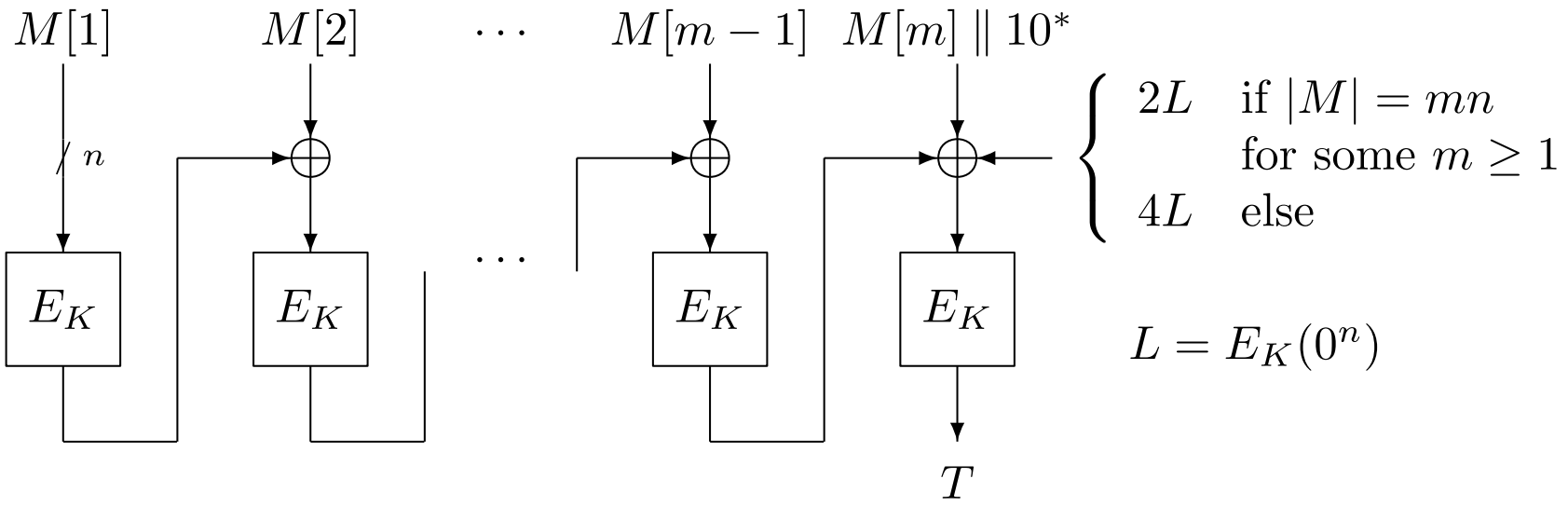
$$\mathbf{Adv}^{\mathrm{prf}}_{\mathrm{CMAC}[\mathrm{AES}]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{AES}}(\mathcal{A}') + \frac{4\sigma^2}{2^{128}}$$

- If E = Rijndael-256

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathrm{CMAC}[\text{Rijndael-256}]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{4\sigma^2}{2^{256}}$$

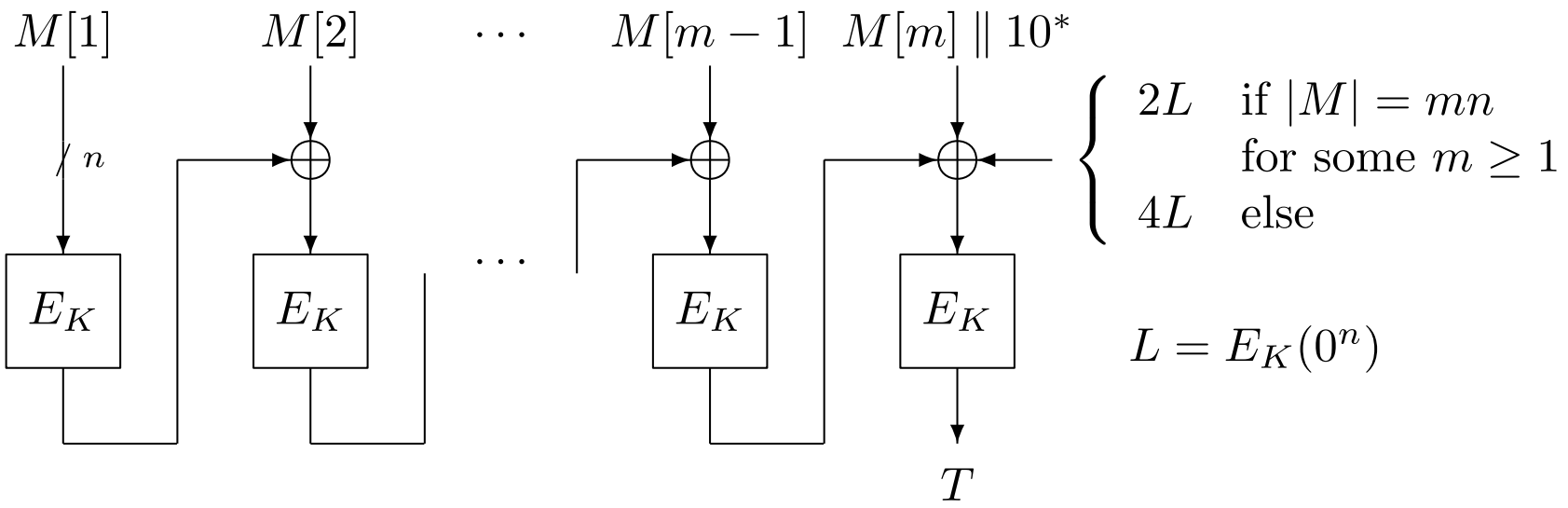
CMAC with AES

- $E = \text{AES}, n = 128$
- $2L$ and $4L$ are computed over $\text{GF}(2^{128})$



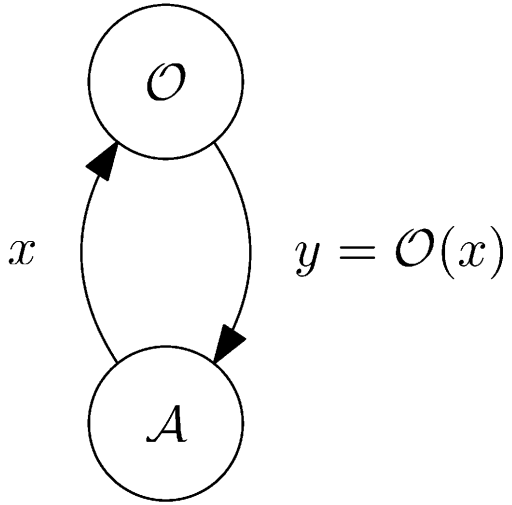
CMAC with Rijndael-256

- E = Rindael-256, $n = 256$
- $2L$ and $4L$ are computed over GF(2^{256})



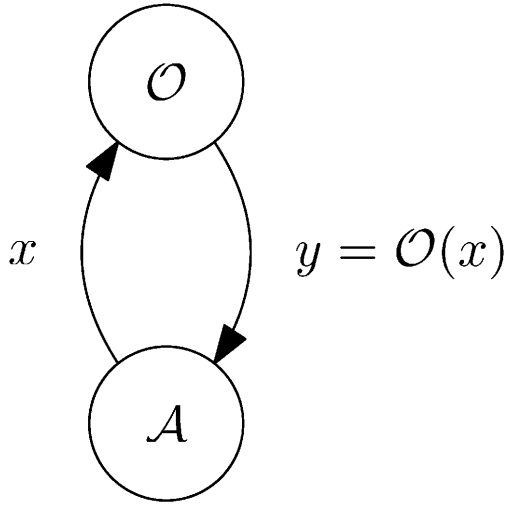
Quantum Attacks

Classical model



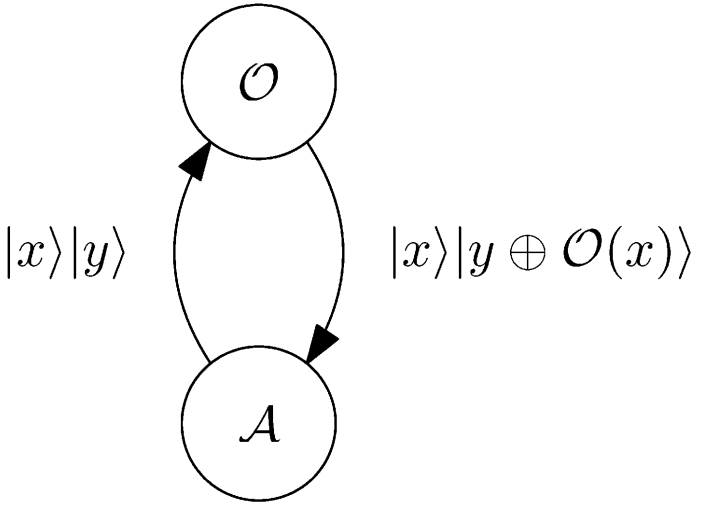
Classical computer

Q1 model



Quantum computer

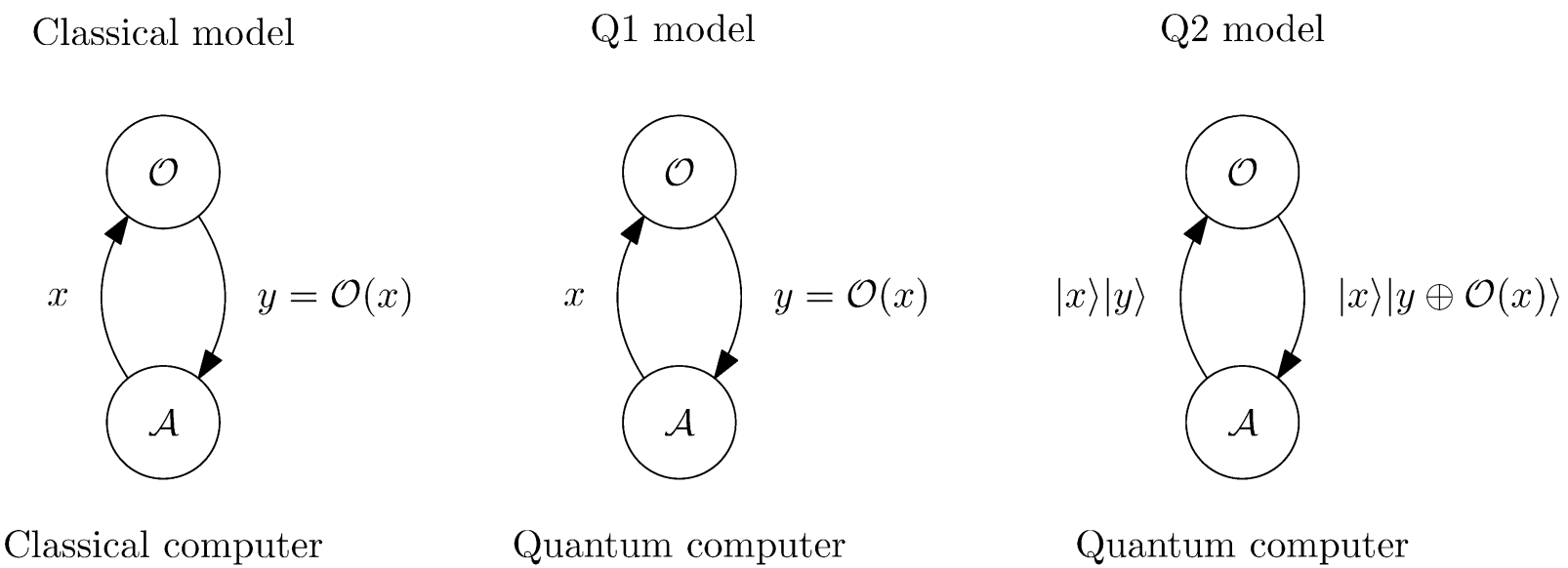
Q2 model



Quantum computer

Quantum Attacks

- CMAC is insecure in Q2 model [KLL+16, SS17]
  - A periodic function can be defined for CMAC, sufficient for existential forgery

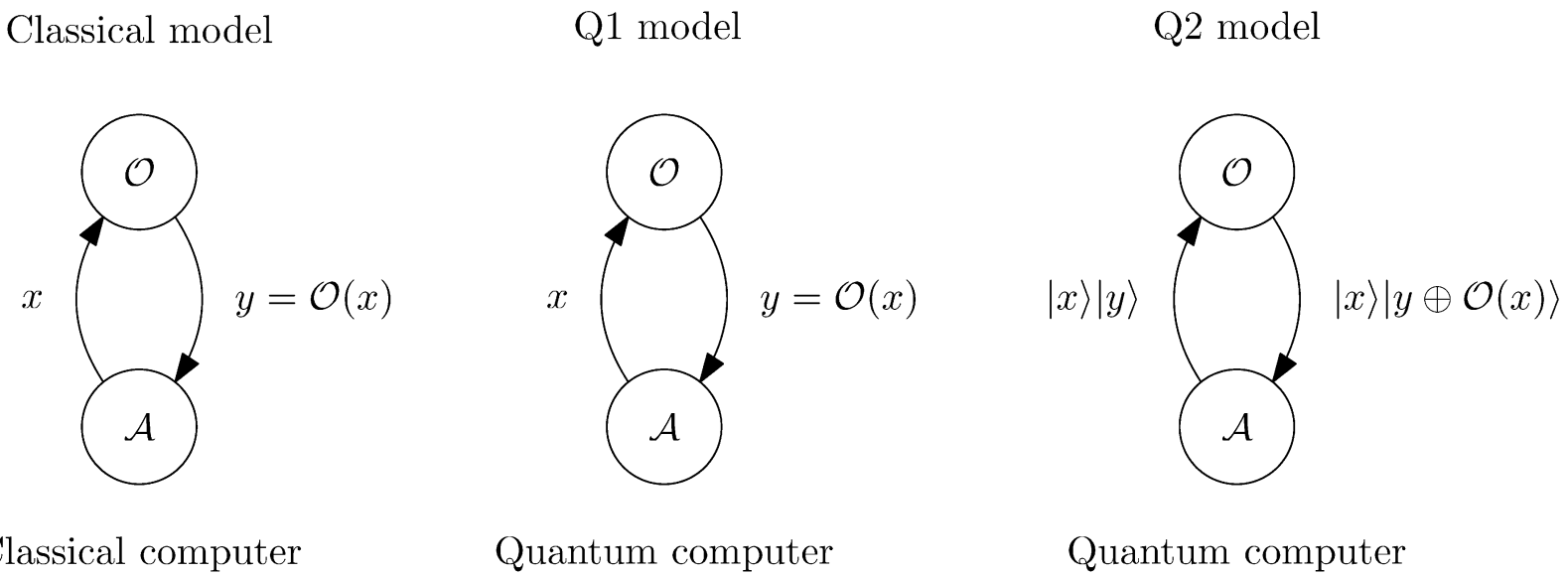


[KLL+16] Kaplan, Leurent, Leverrier, Naya-Plasencia. Breaking symmetric cryptosystems using quantum period finding. CRYPTO 2016.

[SS17] Santoli and Schaffner. Using Simon's algorithm to attack symmetric-key cryptographic primitives. Quant. Inf. Comput., 2017.

Quantum Attacks

- CMAC is insecure in Q2 model [KLL+16, SS17]
- However, in Q1 model, CMAC maintains provable security up to the birthday bound, assuming that the block cipher is secure in Q1 model



[KLL+16] Kaplan, Leurent, Leverrier, Naya-Plasencia. Breaking symmetric cryptosystems using quantum period finding. CRYPTO 2016.

[SS17] Santoli and Schaffner. Using Simon's algorithm to attack symmetric-key cryptographic primitives. Quant. Inf. Comput., 2017.

Security of CMAC in Q1 Model

- CMAC is insecure in Q2 model
- However, in Q1 model, CMAC maintains provable security up to the birthday bound, assuming that the block cipher is secure in Q1 model, following [MS17]

$$\mathbf{Adv}^{\text{q1-prf}}_{\text{CMAC}[E]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{q1-prp}}_{E}(\mathcal{A}') + \frac{4\sigma^2}{2^n}$$

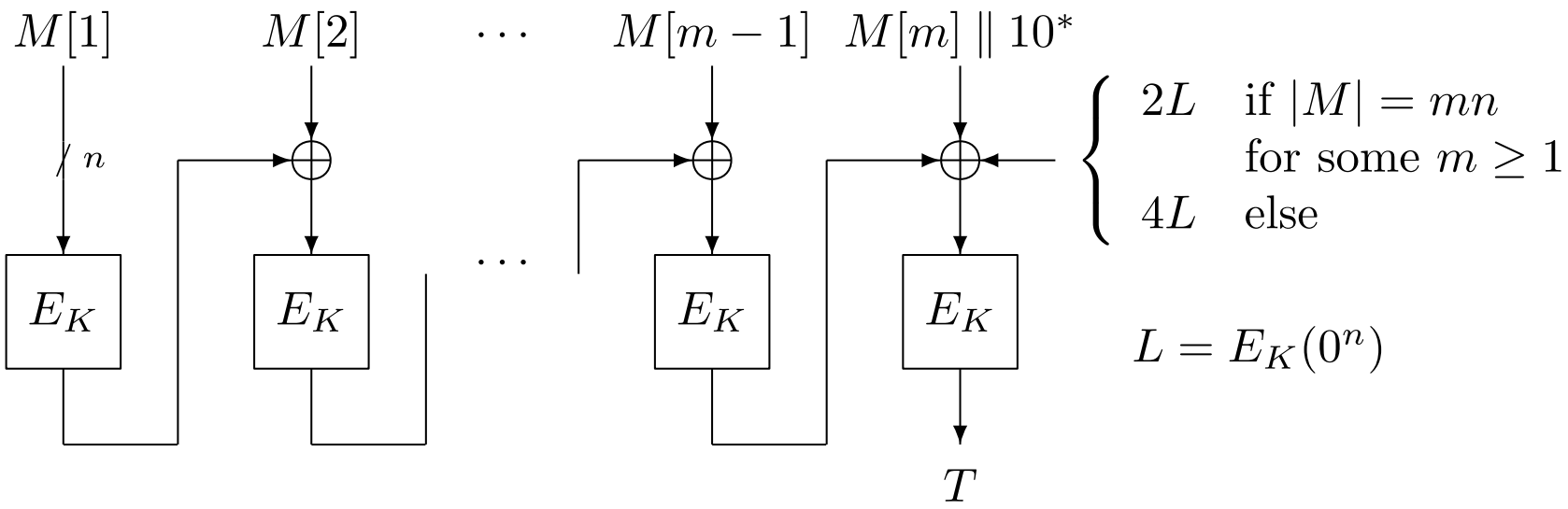
$$\mathbf{Adv}^{\text{q1-prf}}_{\text{CMAC}[\text{AES}]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{q1-prp}}_{\text{AES}}(\mathcal{A}') + \frac{4\sigma^2}{2^{128}}$$

$$\mathbf{Adv}^{\text{q1-prf}}_{\text{CMAC}[\text{Rijndael-256}]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{q1-prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{4\sigma^2}{2^{256}}$$

[MS17] Mennink and Szepieniec. XOR of PRPs in a Quantum World. PQCrypto 2017.

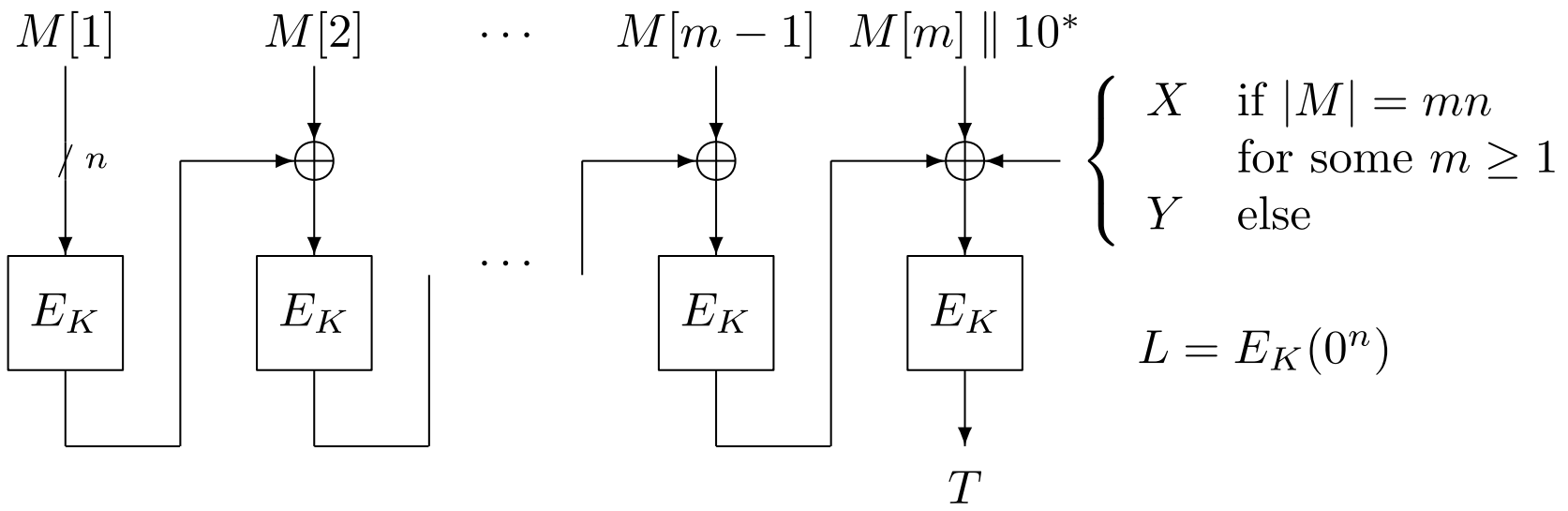
CMAC with Rijndael-256

- E = Rindael-256, $n = 256$
- $2L$ and $4L$ are computed over GF(2^{256})



CMAC with Rijndael-256

- Let $X = 2L$ and $Y = 4L$



CMAC with Rijndael-256

- Conditions on X and Y
  - For any c, if L is chosen uniformly at random,

$$\begin{cases} \Pr[X = c] \leq \epsilon \\ \Pr[Y = c] \leq \epsilon \\ \Pr[X \oplus Y = c] \leq \epsilon \\ \Pr[X \oplus L = c] \leq \epsilon \\ \Pr[Y \oplus L = c] \leq \epsilon \\ \Pr[X \oplus Y \oplus L = c] \leq \epsilon \end{cases}$$

- These six conditions are necessary and sufficient for CMAC to be secure [Iwa03]

[Iwa03] Tetsu Iwata. Six conditions in OMAC-family are tight, Rump session talk at FSE 2003.

CMAC with Rijndael-256

- With $X = 2L$ and $Y = 4L$,

$$\begin{cases} \Pr[X = c] \leq \epsilon \\ \Pr[Y = c] \leq \epsilon \\ \Pr[X \oplus Y = c] \leq \epsilon \\ \Pr[X \oplus L = c] \leq \epsilon \\ \Pr[Y \oplus L = c] \leq \epsilon \\ \Pr[X \oplus Y \oplus L = c] \leq \epsilon \end{cases} \Rightarrow \begin{cases} \Pr[2L = c] \leq \epsilon \\ \Pr[4L = c] \leq \epsilon \\ \Pr[6L = c] \leq \epsilon \\ \Pr[3L = c] \leq \epsilon \\ \Pr[5L = c] \leq \epsilon \\ \Pr[7L = c] \leq \epsilon \end{cases}$$

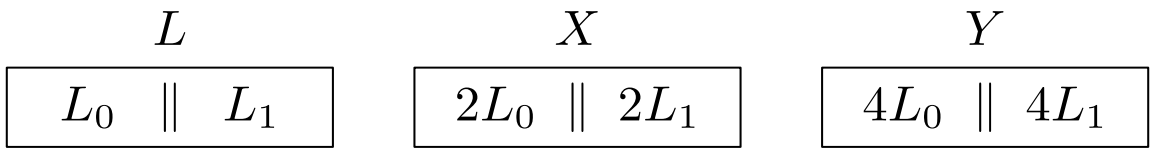
where $\epsilon = 1/2^n$

CMAC with Rijndael-256

- $X = 2L$ and $Y = 4L$ over GF(2^{256}) give $\epsilon = 1/2^{256}$ and work fine
  - Natural option for CMAC with Rijndael-256
- There are many other options that work

CMAC with Rijndael-256

- $L = E_K(0^{256}) = L_0 \parallel L_1 \in \{0,1\}^{128} \times \{0,1\}^{128}$
- $X = 2L_0 \parallel 2L_1$, $Y = 4L_0 \parallel 4L_1$ over GF(2^{128})
- This gives $\epsilon = 1/2^{256}$ and works



$$\begin{cases} \Pr[2L = c] \leq \epsilon \\ \Pr[4L = c] \leq \epsilon \\ \Pr[6L = c] \leq \epsilon \\ \Pr[3L = c] \leq \epsilon \\ \Pr[5L = c] \leq \epsilon \\ \Pr[7L = c] \leq \epsilon \end{cases}$$

CMAC with Rijndael-256

- $L = E_K(0^{256}) = L_0 \parallel L_1 \in \{0,1\}^{128} \times \{0,1\}^{128}$
- $X = 2L_0 \parallel 2L_1$, $Y = 4L_0 \parallel 4L_1$ over GF(2^{128})
- This gives $\epsilon = 1/2^{256}$ and works

L	X	Y
$L_0 \parallel L_1$	$2L_0 \parallel 2L_1$	$4L_0 \parallel 4L_1$

$$\begin{cases} \Pr[2L = c] \leq \epsilon \\ \Pr[4L = c] \leq \epsilon \\ \Pr[6L = c] \leq \epsilon \\ \Pr[3L = c] \leq \epsilon \\ \Pr[5L = c] \leq \epsilon \\ \Pr[7L = c] \leq \epsilon \end{cases}$$

- L can be broken into smaller words
- $L = L_0 \parallel L_1 \parallel L_2 \parallel L_3 \in (\{0,1\}^{64})^4$
- $X = 2L_0 \parallel 2L_1 \parallel 2L_2 \parallel 2L_3$, $Y = 4L_0 \parallel 4L_1 \parallel 4L_2 \parallel 4L_3$ over GF(2^{64})
- This gives $\epsilon = 1/2^{256}$ and works

CMAC with Rijndael-256

- Consider the case that L is broken into 64-bit words
  - $L = L_0 \parallel L_1 \parallel L_2 \parallel L_3 \in (\{0,1\}^{64})^4$
- Let $L_{[0..3]} = L_0 \oplus L_1 \oplus L_2 \oplus L_3$
- $X = L_1 \parallel L_2 \parallel L_3 \parallel L_{[0..3]}$, $Y = L_2 \parallel L_3 \parallel L_{[0..3]} \parallel L_0$
- This works, using only XOR and word shift

CMAC with Rijndael-256

$$\begin{cases} X = L_1 \parallel L_2 \parallel L_3 \parallel L_{[0..3]} \\ Y = L_2 \parallel L_3 \parallel L_{[0..3]} \parallel L_0 \end{cases}$$

$$\begin{cases} X = \begin{bmatrix} L_0 & L_1 & L_2 & L_3 \end{bmatrix} \cdot M_X \\ Y = \begin{bmatrix} L_0 & L_1 & L_2 & L_3 \end{bmatrix} \cdot M_Y \end{cases}$$

$$\text{where} \quad M_X = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad M_Y = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

CMAC with Rijndael-256

X	M_X
Y	M_Y
$X \oplus Y$	$M_X \oplus M_Y$
$X \oplus L$	$M_X \oplus I$
$Y \oplus L$	$M_Y \oplus I$
$X \oplus Y \oplus L$	$M_X \oplus M_Y \oplus I$

$$\begin{cases} \Pr[X = c] \leq \epsilon \\ \Pr[Y = c] \leq \epsilon \\ \Pr[X \oplus Y = c] \leq \epsilon \\ \Pr[X \oplus L = c] \leq \epsilon \\ \Pr[Y \oplus L = c] \leq \epsilon \\ \Pr[X \oplus Y \oplus L = c] \leq \epsilon \end{cases}$$

$$M_X \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad M_Y \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad M_X \oplus M_Y \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad M_X \oplus L \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad M_Y \oplus L \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \quad M_X \oplus M_Y \oplus L \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- All six matrices are full rank
- For each condition, one value of L satisfies the equality, $\epsilon = 1/2^{256}$

CMAC with Rijndael-256

- More options

$$L = L_0 \parallel L_1 \parallel L_2 \parallel L_3 \in \{0,1\}^{256}, L_i \in \{0,1\}^{64}, L_{[a,b]} = L_a \oplus L_b$$

$$\begin{cases} X = L_2 \parallel L_{[2,3]} \parallel L_1 \parallel L_0 \\ Y = L_3 \parallel L_2 \parallel L_{[0,1]} \parallel L_1 \end{cases}$$

CMAC with Rijndael-256

- More options

$$L = L_0 \parallel L_1 \parallel \cdots \parallel L_7 \in \{0,1\}^{256}, L_i \in \{0,1\}^{32}$$

$$\begin{cases} X = L_1 \parallel L_2 \parallel L_3 \parallel L_{[0..3]} \parallel L_5 \parallel L_6 \parallel L_7 \parallel L_{[4..7]} \\ Y = L_2 \parallel L_3 \parallel L_{[0..3]} \parallel L_0 \parallel L_6 \parallel L_7 \parallel L_{[4..7]} \parallel L_4 \end{cases}$$

$$\begin{cases} X = L_2 \parallel L_{[2,3]} \parallel L_1 \parallel L_0 \parallel L_6 \parallel L_{[6,7]} \parallel L_5 \parallel L_4 \\ Y = L_3 \parallel L_2 \parallel L_{[0,1]} \parallel L_1 \parallel L_7 \parallel L_6 \parallel L_{[4,5]} \parallel L_5 \end{cases}$$

$$\begin{cases} X = L_1 \parallel L_2 \parallel L_3 \parallel L_4 \parallel L_5 \parallel L_6 \parallel L_7 \parallel L_{[0,1]} \\ Y = L_2 \parallel L_3 \parallel L_4 \parallel L_5 \parallel L_6 \parallel L_7 \parallel L_{[0,1]} \parallel L_{[1,2]} \end{cases}$$

CMAC with Rijndael-256

- It can be naturally defined
- The security follows from existing works
  - Birthday bound secure in the classical setting (128-bit secure)
  - Birthday bound secure in the Q1 setting (128-bit secure, assuming that Rijndael-256 is Q1 secure)
  - Insecure in the Q2 setting (irrelevant to the security of Rijndael-256)
- Using $X = 2L$ and $Y = 4L$ over $\mathrm{GF}(2^{256})$ works fine, but the definition of the mask could be explored

Outline

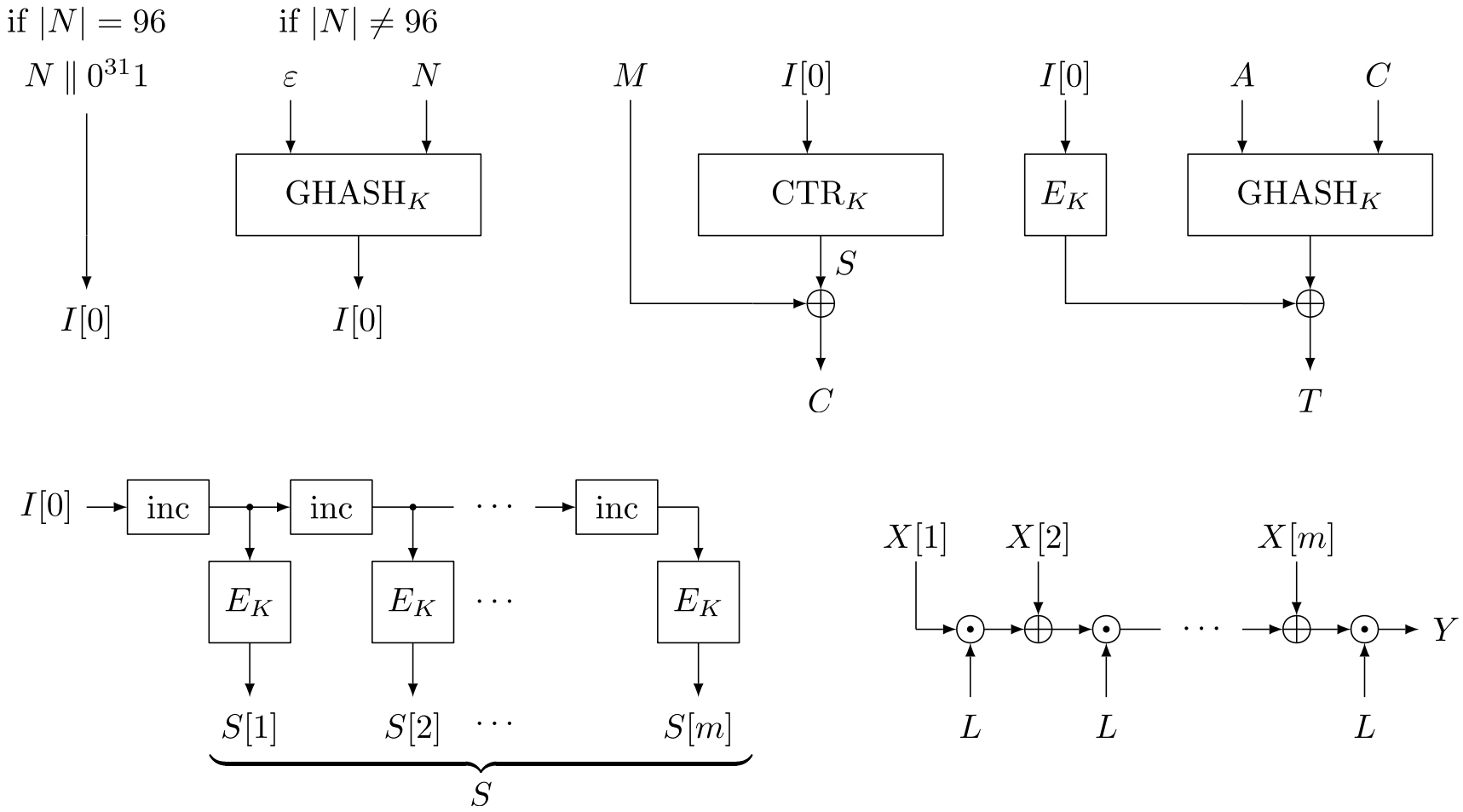
- AES-CMAC and Rijndael-256-CMAC
- AES-GCM and Rijndael-256-GCM

AES-GCM, Rijndael-256-GCM

- GCM, Galois/Counter Mode, an authenticated encryption with associated data (AEAD) scheme for 128-bit block ciphers
- Designed by McGrew and Viega in 2004 [MV04]
- Selected as the NIST recommended authenticated encryption mode in 2007 in NIST SP 800-38D
- Widely used in practice
  - ISO/IEC 19772, IEEE P1619.1, NSA Suite B, IETF IPsec, SSH, SSL,...

[MV04] David A. McGrew and John Viega. The Security and Performance of the Galois/Counter Mode (GCM) of Operation. Cryptology ePrint Archive: Report 2004/193 (full version of INDOCRYPT 2004).

Overview of GCM



Security Analysis

- [Ferguson, ’05]
  - Forgery attacks when the tag is short
- [Joux, ’06]
  - Key recovery attacks on GCM (nonce reuse), forgery
  - Attacks on the draft NIST version of GCM
- [Handschuh, Preneel, ’08]
  - A weak key, forgery attacks
- [Saarinen, ’12]
  - Many weak keys, forgery attacks (cycling attack)
- [Procter, Cid, ’15]
  - Generalization weak key classes and of the cycling attack

Provable Security

- [McGrew, Viega, '04]
  - Initial provable security proof
- [Iwata, Ohashi, Minematsu, '12]
  - Pointed out a gap in the proof of [McGrew, Viega '04]
  - Corrected proofs
    - separating the case for $|N| = 96$ and $|N| \neq 96$, with security bounds containing 2^{22} for the latter
- [Niwa, Ohashi, Minematsu, Iwata, '15]
  - Improved provable security bound for $|N| \neq 96$, reducing 2^{22} to 2^5
- [Hoang, Tessaro, Thiruvengadam, '18]
  - Multi-user security in the ideal cipher model

Provable Security Bounds in [IOM12] and [NOMI15]

$$[\mathrm{IOM12}] \quad |N| = 96, \quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM}[E,\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^n}$$

$$[\mathrm{IOM12}] \quad |N| \neq 96, \quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM}[E,\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^n} + \frac{2^{22} q(\sigma + q)(\ell_N + 1)}{2^n}$$

$$[\mathrm{NOMI15}] \quad |N| \neq 96, \quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM}[E,\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^n} + \frac{2^{5} q(\sigma + q)(\ell_N + 1)}{2^n}$$

Provable Security Bounds in [IOM12] and [NOMI15]

$$\text{[IOM12]}\quad |N| = 96,\quad \mathbf{Adv}^{\text{auth}}_{\text{GCM}[E,\tau]}(\mathcal{A}) \le \mathbf{Adv}^{\text{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^n} + \frac{q'(\ell_A + 1)}{2^\tau}$$

$$\text{[IOM12]}\quad |N| \neq 96,\quad \mathbf{Adv}^{\text{auth}}_{\text{GCM}[E,\tau]}(\mathcal{A}) \le \mathbf{Adv}^{\text{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^n} + \frac{2^{22}(q + q')(\sigma + q + 1)(\ell_N + 1)}{2^n} + \frac{q'(\ell_A + 1)}{2^\tau}$$

$$\text{[NOMI15]}\quad |N| \neq 96,\quad \mathbf{Adv}^{\text{auth}}_{\text{GCM}[E,\tau]}(\mathcal{A}) \le \mathbf{Adv}^{\text{prp}}_{E}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^n} + \frac{2^{5}(q + q')(\sigma + q + 1)(\ell_N + 1)}{2^n} + \frac{q'(\ell_A + 1)}{2^\tau}$$

Provable Security Bounds in [IOM12] and [NOMI15]

- These bounds cover $E = \mathrm{AES}$ $(n = 128)$
- For AES:

$$[\mathrm{IOM12}] \quad |N| = 96, \quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM}[\mathrm{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{AES}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{128}}$$

$$[\mathrm{NOMI15}] \quad |N| \neq 96, \quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM}[\mathrm{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{AES}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{128}} + \frac{2^5 q(\sigma + q)(\ell_N + 1)}{2^{128}}$$

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$$[\mathrm{NOMI15}] \quad |N| \neq 96, \quad \mathbf{Adv}^{\mathrm{auth}}_{\mathrm{GCM}[\mathrm{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{AES}}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^{128}} + \frac{2^5 (q + q')(\sigma + q + 1)(\ell_N + 1)}{2^{128}} + \frac{q'(\ell_A + 1)}{2^{\tau}}$$

- Also valid in Q1 setting

Provable Security Bounds in [IOM12] and [NOMI15]

- For Rijdael-256:

$$[\mathrm{IOM12}]\quad |N| = 96,\quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM[Rijndael\text{-}256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{Rijndael\text{-}256}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{256}}$$

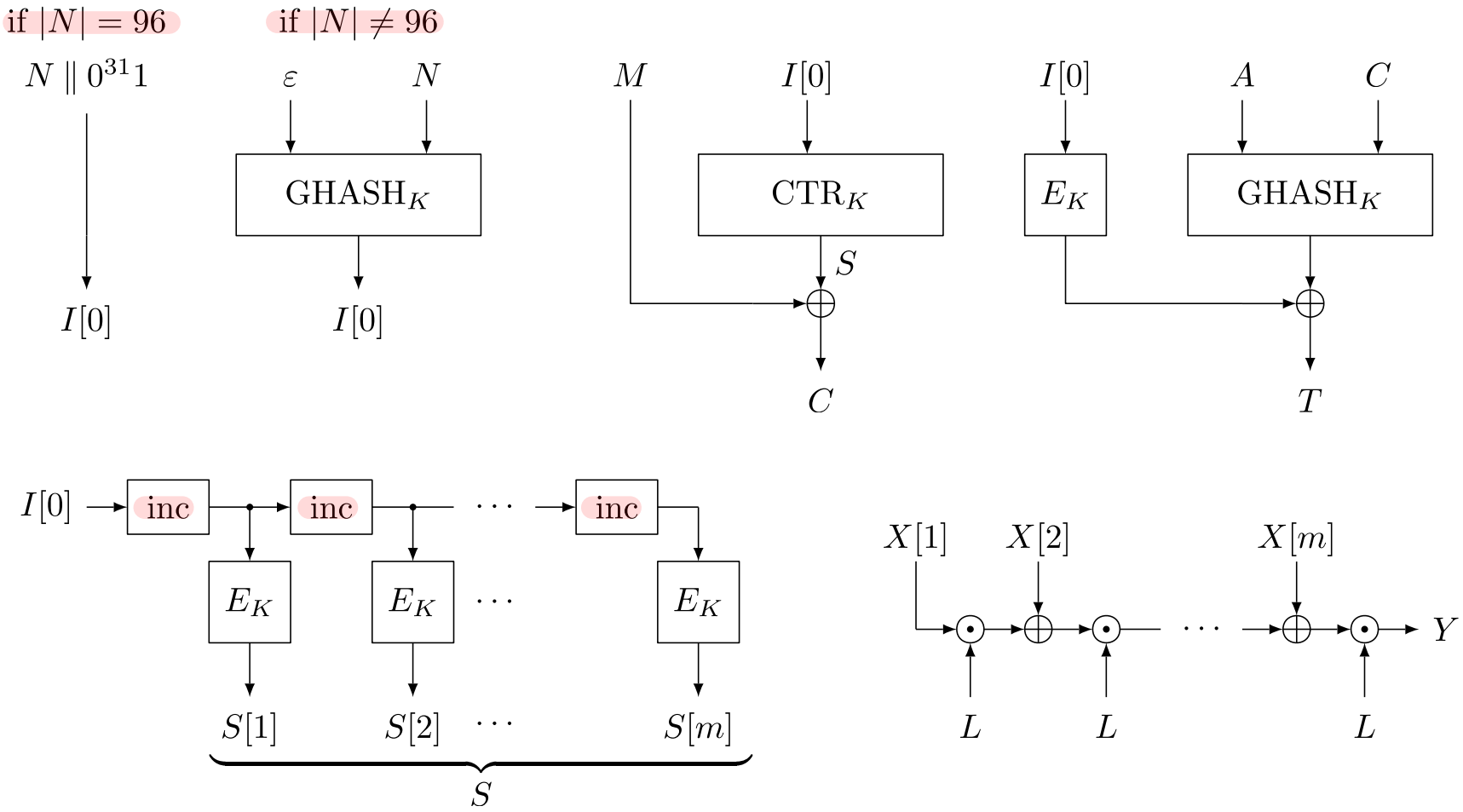
$$[\mathrm{NOMI15}]\ |N| \neq 96,\quad \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{GCM[Rijndael\text{-}256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{Rijndael\text{-}256}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{256}} + \frac{2^5 q(\sigma + q)(\ell_N + 1)}{2^{256}}$$

$$[\mathrm{IOM12}]\quad |N| = 96,\quad \mathbf{Adv}^{\mathrm{auth}}_{\mathrm{GCM[Rijndael\text{-}256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{Rijndael\text{-}256}}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^{256}} + \frac{q'(\ell_A + 1)}{2^{\tau}}$$

$$[\mathrm{NOMI15}]\ |N| \neq 96,\quad \mathbf{Adv}^{\mathrm{auth}}_{\mathrm{GCM[Rijndael\text{-}256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathrm{Rijndael\text{-}256}}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^{256}} + \frac{2^5 (q + q')(\sigma + q + 1)(\ell_N + 1)}{2^{256}} + \frac{q'(\ell_A + 1)}{2^{\tau}}$$

- 96 is used as an absolute value, and inc increments the last 32 bits

Overview of GCM



How Do We Define Rijndael-256-GCM?

- [SP 800-38D] is for a 128-bit block cipher
- [MV04] covers an n-bit block cipher with $n \geq 64$ (the main focus seems to be $n = 128$ and 64)
  - $I[0] = N \parallel 0^{31}1$ if $|N| = n - 32$, $I[0] = \mathrm{GHASH}_L(\varepsilon, N)$ otherwise
  - inc increments the last 32 bits
  - Input of GHASH:
    $X[1], \ldots, X[m] = A[1] \parallel \cdots \parallel A[a]0^* \parallel C[1] \parallel \cdots \parallel C[c]0^* \parallel \mathrm{len}_{n/2}(A) \parallel \mathrm{len}_{n/2}(C)$
  - $Y = X[1]L^m \oplus X[2]L^{m-1} \oplus \cdots \oplus X[m]L$ over $\mathrm{GF}(2^n)$
  - With $n = 256$, the generation of $I[0]$ depends on whether $|N| = 224$ or not
  - One plaintext length is limited to $2^{32} - 2$ blocks (in a 256-bit block), not enough

How Do We Define Rijndael-256-GCM?

- [MV04]
  - $I[0] = N \parallel 0^{31}1$ if $|N| = n - 32$, $I[0] = \mathrm{GHASH}_L(\varepsilon, N)$ otherwise
  - inc increments the last 32 bits
- [IOM12, NOMI15]
  - $I[0] = N \parallel 0^{31}1$ if $|N| = 96$, $I[0] = \mathrm{GHASH}_L(\varepsilon, N)$ otherwise
  - inc increments the last 32 bits
- [AY12]
  - $I[0] = N \parallel 0^{31}1$ if $|N| = 3n/4$, $I[0] = \mathrm{GHASH}_L(\varepsilon, N)$ otherwise
  - inc increments the least $n/4$ bits
  - The parameter choice of [AY12] allows 192-bit nonces and inc works on 64 bits for $n = 256$

[AY12] Aoki, Yasuda. The Security and Performance of "GCM" when Short Multiplications Are Used Instead. Inscrypt 2012

Provable Security Bounds in [IOM12] and [NOMI15]

- For Rijdael-256:

$$[\text{IOM12}]\quad |N| = 96,\quad \mathbf{Adv}^{\text{priv}}_{\text{GCM[Rijndael-256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{256}}$$

$$[\text{NOMI15}]\ |N| \neq 96,\quad \mathbf{Adv}^{\text{priv}}_{\text{GCM[Rijndael-256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{0.5(\sigma + q + 1)^2}{2^{256}} + \frac{2^5 q(\sigma + q)(\ell_N + 1)}{2^{256}}$$

$$[\text{IOM12}]\quad |N| = 96,\quad \mathbf{Adv}^{\text{auth}}_{\text{GCM[Rijndael-256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^{256}} + \frac{q'(\ell_A + 1)}{2^{\tau}}$$

$$[\text{NOMI15}]\ |N| \neq 96,\quad \mathbf{Adv}^{\text{auth}}_{\text{GCM[Rijndael-256},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{Rijndael-256}}(\mathcal{A}') + \frac{0.5(\sigma + q + q' + 1)^2}{2^{256}} + \frac{2^5(q + q')(\sigma + q + 1)(\ell_N + 1)}{2^{256}} + \frac{q'(\ell_A + 1)}{2^{\tau}}$$

- 96 can be changed to 192. Changing inc to increment the last 64 bits does not affect [IOM12], $|N| = 192$, but [NOMI15] bounds should be reviewed

GHASH in Rijndael-256-GCM

- A naïve definition of GHASH works in $\mathrm{GF}(2^{256})$
- $X[1], \ldots, X[m] = A[1] \| \cdots \| A[a]0^* \| C[1] \| \cdots \| C[c]0^* \| \mathrm{len}_{128}(A) \| \mathrm{len}_{128}(C)$
- $Y = X[1]L^m \oplus X[2]L^{m-1} \oplus \cdots \oplus X[m]L$ over $\mathrm{GF}(2^{256})$

- [AY12] proposes to use a multiplication over $\mathrm{GF}(2^{64})$ to define GHASH over 128-bit blocks
  - Called GCM/2+
  - Can be used to define GHASH over 256-bit blocks based on a multiplication over $\mathrm{GF}(2^{128})$, needs modification to other places, but achieves 128-bit security
  - [Procter, Cid, '15] shows the analysis of weak key classes

[AY12] Aoki, Yasuda. The Security and Performance of "GCM" when Short Multiplications Are Used Instead. Inscrypt 2012.

GHASH in Rijndael-256-GCM

- GHASH over $\{0,1\}^{256}$ blocks based on GF(2^{128}) multiplication
- $L = E_K(0^{256}) = L_0 \parallel L_1 \in \{0,1\}^{128} \times \{0,1\}^{128}$
- $X[1], \ldots, X[m] = A[1] \parallel \cdots \parallel A[a]0^* \parallel C[1] \parallel \cdots \parallel C[c]0^* \parallel \mathrm{len}_{128}(A) \parallel \mathrm{len}_{128}(C)$
  - Break into 128-bit blocks
- $Y_0 = X[1]L_0^m \oplus X[2]L_0^{m-1} \oplus \cdots \oplus X[m]L_0$
  $Y_1 = X[1]L_1^m \oplus X[2]L_1^{m-1} \oplus \cdots \oplus X[m]L_1$
  over GF(2^{128})
- $Y = Y_0 \parallel Y_1$

- ϵ-almost XOR universal hash function with $\epsilon = \ell^2/2^{256}$

GHASH in Rijndael-256-GCM

- Focusing on the case $|N|$ fixed, with Rijdael-256,

$$[\text{IOM12}] \quad \mathbf{Adv}^{\text{priv}}_{\text{GCM}[\text{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{A}') + \frac{0.5(\sigma+q+1)^2}{2^{128}}$$

$$\Longrightarrow \quad \mathbf{Adv}^{\text{priv}}_{\text{GCM}[\text{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{A}') + \frac{0.5(\sigma+q+1)^2}{2^{128}} \qquad (\text{unaffected})$$

$$[\text{IOM12}] \quad \mathbf{Adv}^{\text{auth}}_{\text{GCM}[\text{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{A}') + \frac{0.5(\sigma+q+q'+1)^2}{2^{128}} + \frac{q'(\ell_A+1)}{2^{\tau}}$$

$$\Longrightarrow \quad \mathbf{Adv}^{\text{auth}}_{\text{GCM}[\text{AES},\tau]}(\mathcal{A}) \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{A}') + \frac{0.5(\sigma+q+q'+1)^2}{2^{128}} + \frac{q'(\ell_A+1)^2}{2^{\tau}}$$

- The bound is still good when $\tau = 256$

GCM with Rijndael-256

- GCM with Rijndael-256 can be defined naively, but the several parameter choices can be reviewed
  - Whether $|N| = 192$ or not, or whether $|N| = 224$ or not, fixed length, or if other lengths are allowed
  - Whether ctr works on the last 32 bits, or 64 bits, or other length
    - Affects the maximum number of plaintext blocks
  - The provable security bounds in [IOM12], $|N| = 192$, is unaffected by the length of inc, but it affects the bounds in [NOMI15], $|N| \neq 192$
- Use of a 256-bit output universal hash function defined over GF(2^{128}) could be explored more

Summary

- Rijndael-256-CMAC can be naturally defined
- The security follows from existing works
- Using $X = 2L$ and $Y = 4L$ over GF(2^{256}) works fine, but the definition of the mask could be explored

- Rijndael-256-GCM can also be defined naturally, but several parameter choices can be reviewed
  - The nonce length
  - The inc function
- Use of a 256-bit output universal hash function defined over GF(2^{128}) could be explored more